Cylinders Solution

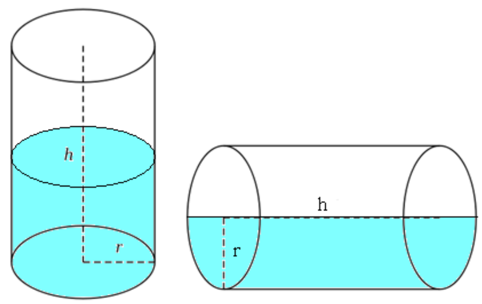
1. The wording makes the question harder than it actually is.

   The question basically asks: if we have a cylindrical tank of radius R and height H, then which of the following changes will give a cylindrical tank with the volume with greatest difference from the volume of $2\pi r^2h$ (twice the original).

   Answer E ($\pi(1.5r)^2*(0.8h)=1.8\pi r^2h$) gives the greatest difference from $2\pi r^2h$.

2. Look at the diagram below.

   Since the tank is half full when placed upright then naturally it'll also be half full when placed on its side, so the level of the water (when placed that way) will be half of the diameter, so r.

   Now, given that $V_{water}=\pi*r^2*H_{water}$ --> $36\pi=\pi r^2*4$ --> $r=3$.

   Answer: B.

3. Diameter of can = 4
   We can fill max. cans when the height is 14 and the base is 20 by 16.
   This is because:
   if base is 14 by 16 then we can fit a max of 14/4 * 16/4 = 12 cans.
   if base is 14 by 20 then we can fit a max of 14/4 * 20/4 = 15 cans.
   if base is 20 by 16 or 16 by 20 then we can fit a max of 20/4 * 16/4 = 20 cans.

4. $volume_{cylinder}=\pi r^2h$

   If the cylinder is placed on 8*10 face then it's maximum radius is 8/2=4 and $volume==\pi*4^2*12=196\pi$;
   If the cylinder is placed on 8*12 face then it's maximum radius is 8/2=4 and $volume==\pi*4^2*10=160\pi$;
   If the cylinder is placed on 10*12 face then it's maximum radius is 10/2=5 and $volume==\pi*5^2*8=200\pi$;

   So, the maximum volume is for 200π.

   Answer: B.

5. Volume of the cylinder equals to $area = \pi r^2 h$. First of all note that answer choices C, D, and E don't make sense. For example cylinder of a radius 6 (option C) just won't fit on any face, as max face has a dimensions 12*10 so cylinder with max radius of 5 can be placed on it.

   Max volume will be when the base of a cylinder is placed on the face with dimension 12*10 thus the radius will be 5 --> $v = \pi 5^2 * 8 = 200\pi$;

   Other options:
   If we place cylinder on the face with dimension 12*8 then radius will be 4 and $v = \pi 4^2 * 10 = 160\pi$;
   If we place cylinder on the face with dimension 10*8 then radius will be 4 and $v = \pi 4^2 * 12 = 192\pi$,.

   Answer: B.

6. $volume_{cylinder} = \pi r^2 h$

   If the cylinder is placed on 6*8 face then it's maximum radius is 6/2=3 and $volume == \pi * 3^2 * 10 = 90\pi$;
   If the cylinder is placed on 6*10 face then it's maximum radius is 6/2=3 and $volume == \pi * 3^2 * 8 = 72\pi$;
   If the cylinder is placed on 8*10 face then it's maximum radius is 8/2=4 and $volume == \pi * 4^2 * 6 = 96\pi$;

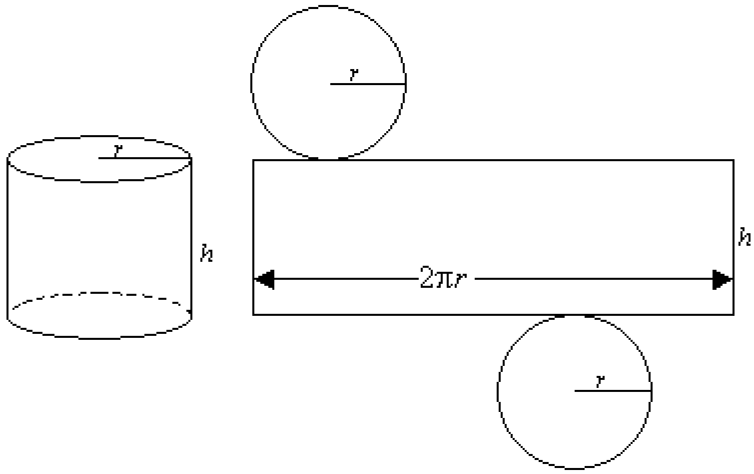
   So, the maximum volume is for $r = 4$.

   Answer: B.

7. "A rectangular steel container has three equal dimensions of 8 inches" means that the container is a cube. Now, cylinder to have maximum possible volume it should have maximum possible diameter and height, so a diameter equal to a side of the cube, so d=8 (radius=4) and a height also equal to a side of the cube, so height=8: $volume_{cylinder} = \pi r^2 h = 128\pi$.

   Answer: C.

8. The surface area without the top and the bottom



is $circumference*height = 2\pi rh = 96\pi$;

9. We are basically told that a cylinder with a height of 0.7 (7/10) meters has the volume of 22 cubic meters.

$volume_{cylinder} = \pi r^2 h = 22$ --> $\pi \approx \frac{22}{7}$ --> $\frac{22}{7} * r2 * \frac{7}{10} = 22$ --> $r = \sqrt{10}$.

Answer: B.

10. Initial volume = (3/4)×∏×5²×8 = 150∏
Relative Drain/ min = .08∏ - .03∏ = .05∏ m^3/min drain
Relative drain / hour = .05∏×60 = 3∏ m^3/ hr

Every one hour starting from 1pm, 4∏ m^3 of water is drained. It means that only at the hour the water is drained and NOT "in that 1 hour"

So after 1 hr the relative drain would be 3∏ + 4∏ = 7∏m^3 of water drain

What i did initially was formed an equation 150∏ = 7∏×n (n is the number of hrs) so ended up with 21 3/7. This wrong

Look at this way
after 21 hrs the amount of water drain will be 21×7∏ = 147∏ m^3
Left over water in the tank after 21 hrs = 3∏ m^3
From above we know that it take 1 more hour to drain that 3∏ m^3.

So answer is 22hrs.

11. Say the the width of the strip each eats is x.

    Since Helga eats the corn circularly, then the number of strips she eats is $\frac{h}{x}$.
    Since Rob eats the corn along the height, then the number of strips he eats is $\frac{2\pi r}{x}$.

    We are told that $\frac{h}{x}=\frac{1}{2}*\frac{2\pi r}{x}$ --> $\frac{h}{r}=\pi$.

    Answer: D.

12. Since the tank is half full when placed upright then it will also be half full when placed on its side, so the level of the water will be half of the diameter, so r.

    Now, given that $V_{water}=\pi^*r^2*H_{water}$ --> $20\pi=\pi r^2*5$ --> $r=2$.

    Answer: C.

13. The area of the cylinder is πr^2h. Since given that the diameter equals the height, then 2r=h and the area becomes $2\pi r^3$.

    Now, if we decrease r by 60% it becomes 0.4r and thus the new volume becomes $0.4^3*(2\pi r^3)=0.064*(2\pi r^3)$. Therefore the volume decreased by approximately 1-0.06=0.94=94%.

    Answer: E.

14. Since 36 cubic inches of water occupy 1/2 of the cylinder, then the volume of the cylinder is 72 cubic inches.

    So, we have that $volume_{cylinder}=\pi^*r^2*h=72$ --> $\pi^*r^2*9=72$ --> $r^2=\frac{8}{\pi}$ --> $r=\sqrt{\frac{8}{\pi}}=2^*\sqrt{\frac{2}{\pi}}$. Hence the diameter equals $2^*(2^*\sqrt{\frac{2}{\pi}})=4^*\sqrt{\frac{2}{\pi}}$.

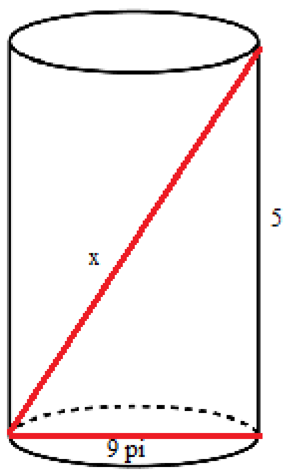
    Answer: E.

15. The volume of the truck's tank is $V=\pi r^2h=\pi 5^2*10=250\pi$. So, 250π cubic feet of water is pumped from the stationary tank.

    What is the height of this amount of water in the stationery tank? $V=\pi r^2h=\pi 100^2*h=250\pi$ --> $h=0.025$ feet.

    Answer: E.

16. Look at the diagram below:



The greatest distance between two points would be x.

Now, since $area = 9\pi = \pi r^2$, then $r = 3$and $d = 6$ --> $x = \sqrt{5^2+6^2} = \sqrt{61}$.

Answer: C.

17. Since the volume of soda in the can is 72π and the can is 3/4 full of soda then the volume of the can is $\frac{3}{4}*can = 72\pi$ --> $can = 96\pi$.

The volume of a cylinder is $\pi r^2 h$ --> $\pi 4^2 h = 96\pi$ --> $h = 6$.

Answer: E.

18. The container shown is a half of a cylinder, with the height of 10 feet and the radius of a base of 8/2=4 feet. Its volume is $\frac{1}{2}\pi r^2 h = 80\pi$ cubic foot.

The weight of the slag is $80\pi * 50 = 4{,}000\pi$ pounds, which is $\frac{4{,}000\pi}{2{,}000} = 2\pi \approx 6.28$ tons.

Answer: C.

19. $Volume_A = \pi r^{2*} h$;
$Volume_B = \pi(3r)^{2*}\frac{h}{3} = 3\pi r^{2*} h$;

Ratio 1:3. Answer is C.

20. The volume of a cylinder is $V = \pi^* r^{2*} h$;
The volume of P is $V_P = \pi^* r^{2*} h$ (where r and h are the radius and the height of P)and the volume of Q is $V_Q = \pi^*(2r)^{2*}\frac{h}{2} = \pi^* 2r^{2*} h$;

Hence $\frac{V_P}{V_Q} = \frac{\pi^* r^{2*} h}{\pi^* 2r^{2*} h} = \frac{1}{2}$.

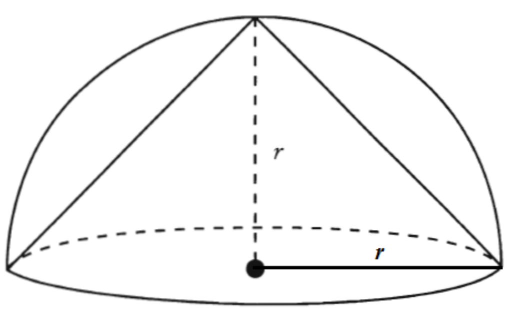
Answer: C.

21. In 10 min the volume of the water which leaked out the cylinder would be $10^* 0.31 = 3.1$ m^3.

Volume = $\pi^* r^{2*} height = 3.1$ --> $\pi^* r^{2*} 0.25 = 3.1$ --> $\pi = 3.14$ --> $3.14^* r^{2*} 0.25 = 3.1$ --> $r = 2$ (approximately).

22. Note that a hemisphere is just a half of a sphere.

Now, since the cone is a right circular one, then the vertex of the cone must touch the surface of the hemisphere directly above the center of the base (as shown in the diagram below), which makes the height of the cone also the radius of the hemisphere. Answer is B.



23. We can make 2 cylinders:

With height of 6 and the radius of the base of $r=\frac{5}{\pi}$ (from $2\pi r=10$ --> $r=\frac{5}{\pi}$) --> $volume=\pi r^2h=\frac{150}{\pi}$.

With height of 10 and the radius of the base of $r=\frac{3}{\pi}$ (from $2\pi r=6$ --> $r=\frac{3}{\pi}$) --> $volume=\pi r^2h=\frac{90}{\pi}$.

The volume of the first one is $\frac{60}{\pi}$ cubic inches greater than the volume of the second one. Answer B.

24. (1) Each of the cans has a radius of 4 centimeters --> radius=4 means that diameter=8, which implies that along the 48 centimeter length of the carton 48/8=6 cans can be placed and along the 32 centimeter width of the carton 32/8=4 cans can be placed. Thus, k=6*4=24. Sufficient.

(2) Six of the cans fit exactly along the length of the carton --> the diameter of the can is 48/6=8 centimeters. So, we have the same info as above. Sufficient.

Answer: D.

25. (1) The interior volume of this carton is 2,304 cubic inches. No information about the cans. Not sufficient.
(2) The exterior of each can is 6 inches high and has a diameter of 4 inches. No information about the cartons. Not sufficient.

(1)+(2) If the dimensions of the carton are 1 by 1 by 2,304, then zero cylindrical cans can be packed in the carton but if the dimensions of the carton are 12 by 12 by 16, then more than zero cylindrical cans can be packed in the carton. Not sufficient.

Answer: E.

26. Notice that the radius of 2 inches means the diameter of 4 inches.

(1) The volume of the box is 240 cubic inches --> if the dimensions are 1*1*240 then we can fit zero cans but if the dimensions are 4*6*10 then we can fit more than zero cans. Not sufficient.

(2) The length of the box is 3 inches --> since both the diameter (4) and height (6) of the cans are greater than the length of the box (3) then zero cans can be fit into the box. Sufficient.

Answer: B.

27. The volume of a cylinder is given by: $volume_{cylinder} = \pi * r^2 * h$.

From (1) we have that t/2=d/3,

thus $volume_A = \pi * (\frac{d}{2})^2 * h$ and $volume_B = \pi * (\frac{t}{2})^2 * 2h = \pi * (\frac{d}{3})^2 * 2h$.

From (2) we have that 1 unit volume of A costs $\frac{1.59}{\pi * (\frac{d}{2})^2 * h} = \frac{4 * 1.59}{\pi d^2 h}$ and 1 unit volume of B costs $\frac{1.39}{\pi * (\frac{d}{3})^2 * 2h} = \frac{4.5 * 1.39}{\pi d^2 h}$. Now, all we need to do is to see which one is less 4*1.59 or 4.5*1.39. Answer is C.

28. Complete solution:

To find the time needed for 30 liters of water to evaporate we need to find the surface area of the top of the cylinder: $2 * area$ will be the amount of water that evaporates each hour, thus $time = \frac{30}{2 * area}$.

On the other hand since $volume = \pi r^2 h = 72$ then $area = \pi r^2 = \frac{72}{h}$. So, basically all we need is ether the area of the surface or the height of the cylinder.

(1) The height of the cylinder is 2 meters. Sufficient.

(2) The radius of the base of the cylinder is $\frac{6}{\sqrt{\pi}}$ meters --> $area = \pi r^2 = 36$. Sufficient.

Answer: D.